2018년 전기 입시 기출문제

2017/8/21~22

1.(15) Consider a linear program: max $c'x$, subject to $Ax \leq b$, where A is an $m \times n$ matrix. Suppose that $x^i \in R^n$, i =1, … , k are distinct optimal solutions to the linear program. Show that the point $x^0 = \sum_{i=1}^{k} \lambda_i x^i$, obtained with some λ_i satisfying $\sum_{i=1}^{k} \lambda_i = 1$, and $\lambda_i \geq 0, i = 1, \ldots, k$, is also an optimal solution.

2.(35) Consider a linear program: max $c'x$, subject to $Ax \leq b, x \geq 0$, where A is a 2× 3 matrix. After adding slack variables x_4 and x_5 to the first and second constraints respectively, some pivoting (not necessarily the simplex pivots) have been performed and the following tableau is obtained. Currently, x_1 and x_3 are basic variables

$$-z \quad - 4x_2 \qquad - x_4 - x_5 = -12$$

$$x_1 - 3x_2 \qquad + 11x_4 - x_5 = -4$$

$$6x_2 + x_3 \quad - 3x_4 + 2x_5 = 3$$

$$x_j \geq 0 \quad \text{for all } j$$

(a)(15) Identify the dual solution y_1, y_2 corresponding to the current basis. Is the solution feasible to the dual problem?

(b)(10) Perform one iteration of the dual simplex method on this tableau.

(c)(10) Can you perform the next dual simplex iteration on the tableau obtained in (b)? What can be said about the status of the primal LP? (e.g., finite optimal, unbounded, or infeasible) What about the status of the dual of the problem? You need to state your reasoning.

3. An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel that her claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove her claim, we administer her prescribed dosage to 20 insomniacs and we observe Y, the number for whom the drug dose induces sleep. We wish to test the hypothesis $H_0 : p = 0.8$ versus the alternative, $H_a : p < 0.8$. Assume that the rejection region $\{y \leq 12\}$ is used.

1) In terms of this problem, what is a type I error?

2) Find α. (You can use the attached table)

3) In terms of this problem, what is a type II error?

4) Find β when $p = 0.4$. (You can use the attached table)

Table 1 Binomial Probabilities

Tabulated values are

$P(Y \leq a) = \sum_{y=0}^{a} p(y)$. (Computations are rounded at third decimal place.)

(d) $n = 20$

	p													
a	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99	*a*
0	.818	.358	.122	.012	.001	.000	.000	.000	.000	.000	.000	.000	.000	0
1	.983	.736	.392	.069	.008	.001	.000	.000	.000	.000	.000	.000	.000	1
2	.999	.925	.677	.206	.035	.004	.000	.000	.000	.000	.000	.000	.000	2
3	1.000	.984	.867	.411	.107	.016	.001	.000	.000	.000	.000	.000	.000	3
4	1.000	.997	.957	.630	.238	.051	.006	.000	.000	.000	.000	.000	.000	4
5	1.000	1.000	.989	.804	.416	.126	.021	.002	.000	.000	.000	.000	.000	5
6	1.000	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000	.000	6
7	1.000	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000	.000	7
8	1.000	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000	.000	8
9	1.000	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000	.000	9
10	1.000	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000	.000	10
11	1.000	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000	.000	11
12	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.032	.000	.000	.000	12
13	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000	.000	13
14	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000	.000	14
15	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003	.000	15
16	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016	.000	16
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075	.001	17
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264	.017	18
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642	.182	19

Solution)

Note that Y is binomial with parameters $n = 20$ and p.

1) If the experimenter concludes that less than 80% of insomniacs respond to the drug when actually the drug induces sleep in 80% of insomniacs, a type I error has occurred.
2) $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true}) = P(Y \leq 12 \mid p = .8) = .032$.
3) If the experimenter does not reject the hypothesis that 80% of insomniacs respond to the drug when actually the drug induces sleep in fewer than 80% of insomniacs, a type II error has occurred.
4) $\beta(.4) = P(\text{fail to reject } H_0 \mid H_a \text{ true}) = P(Y > 12 \mid p = .4) = .021$.

2017년 후기 입시 기출문제

2017/5/24

1.(50 pts.) Consider the following primal linear program:

(P) max $4x_1 + x_2 + 5x_3 + 3x_4$

s.t. $x_1 - x_2 - x_3 + 3x_4 \leq 1$

$5x_1 + x_2 + 3x_3 + 8x_4 \leq 55$

$-x_1 + 2x_2 + 3x_3 - 5x_4 \leq 3$

$x_j \geq 0$ for all j

(a)(10) State the dual of the primal linear program (P).

We add slack variables $x_5, x_6,$ and x_7 to the first, second, and third constraint respectively, and simplex method is used to solve the problem. Then, the following tableau is obtained:

$$\begin{array}{ccccccccccc} -z & -x_1 & & -2x_3 & & -11x_5 & & -6x_7 & = & -29 \\ & 2x_1 & +x_2 & +4x_3 & & +5x_5 & & +3x_7 & = & 14 \\ & x_1 & & +x_3 & +x_4 & +2x_5 & & +x_7 & = & 5 \\ & -5x_1 & & -9x_3 & & -21x_5 & +x_6 & -11x_7 & = & 1 \end{array}$$

(b)(20) Identify the current basic feasible solution from this tableau. Verify that the solution is an optimal solution to the primal. (Citing the optimality criterion of the simplex method is not enough. You need to show that satisfying the optimality criterion of the simplex method guarantees the optimality of the current solution.) Is the current optimal solution a unique optimal solution? Explain why or why not

(c)(20) Find an optimal solution to the dual problem and prove that it is an optimal solution to the dual problem. (You may use the weak duality and strong duality theorems without proving them.)

2. Let $X_1, X_2, \ldots, X_n$ denote an independent random sample of size n (>1) from a population with $\mathrm{E(X)} = \mu$ and $\mathrm{Var(X)} = \sigma^2$. Suppose that we have two estimators of μ: X_1 and $\bar{X}$.

(1) Show if both estimator are unbiased estimators of μ.

(2) Calculate the mean squared error (MSE) of both estimators.

(3) Which estimator is preferred to estimate μ and why?

Solution)

(1) $E(X_1) = \mu$ => X_1 is an unbiased estimator.

$E(\bar{X}) = \mu$ => $\bar{X}$ is an unbiased estimator.

(2) $MSE(X_1)=Var(X_1)+bias^2(X_1)= \sigma^2+0^2=\sigma^2$

$MSE(\bar{X})=Var(\bar{X})+bias^2(\bar{X})= \sigma^2/n+0^2=\sigma^2/n$

(3) $\bar{X}$ is preferred due to the smaller MSE.

2017 전기 입시 기출문제

2016/8/9

1.(10 pts) Let $C, D \subseteq R^n$ be convex sets. Define the set $C + D \equiv \{x + y: x \in C, y \in D\}$. Show that $C + D$ is convex.

2.(10 pts) Suppose that f_1, f_2 are convex functions from R^n into R and let $f(x) = f_1(x) + f_2(x)$. Show that f is a convex function.

3. (20 pts) Consider the following pair of constraints:

(I) $\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i, \qquad i = 1, \dots, m$

$x_j \geq 0, \qquad j = 1, \dots, n$

(II) $y_i \geq 0, \qquad i = 1, \dots, m$

$\sum_{i=1}^{m} a_{ij}\, y_i \geq 0, \qquad j = 1, \dots, n$

$\sum_{i=1}^{m} b_i y_i < 0$

Give a proof that system (I) is infeasible if and only if system (II) is feasible. i.e. exactly one of the system (I) and (II) has a feasible solution.

2. Let $\bar{Y}$ be the mean of a random sample of size n=16 from a normal distribution with mean μ and

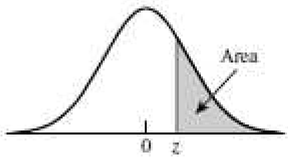
standard deviation σ=8. You want to test $H_0 : \mu = 35$ vs. $H_a : \mu > 35$ by rejecting H_0 when $\bar{Y} > 36.5$. When you answer the questions below, you can use the attached Z-table.

(a) Determine the significance level α of this test.

(b) Suppose that the true value of μ is 36. Determine the power of this test when μ=36.

(c) Suppose that the true value of μ is 36, as in (b). Determine the probability of a Type II error.

Table 4 Normal Curve Areas
Standard normal probability in right-hand tail
(for negative values of *z*, areas are found by symmetry)



	Second decimal place of *z*									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611

==

Solution)

(a) $\bar{Y} \sim N(35, 8^2/16) = N(35, 4)$.

$\alpha = P\{\bar{Y} > 36.5 \mid \mu = 35\} = P\left(\frac{\bar{Y}-35}{\sqrt{4}} > \frac{36.5-35}{\sqrt{4}}\right) = P(Z > 0.75) = 0.2266.$

(b) Power = $P\{\bar{Y} > 36.5 \mid \mu = 36\} = P\left(\frac{\bar{Y}-36}{\sqrt{4}} > \frac{36.5-36}{\sqrt{4}}\right) = P(Z > 0.25) = 0.4013.$

(c) Type II error = 1−power = 1−0.4013 = 0.5987.